N°. VIII.

A Letter from Mr. ANDREW ELLICOTT, *to* Mr. ROBERT PATTERSON.

A Method of Calculating the Eccentric Anomaly of the Planets.

Philadelphia, April 4th, 1794.

SIR,

Read April 4, 1794.

HAVING occafion fome years ago to conftruct a fet of aftronomical tables for the planet ♅, I made ufe of an operation to obtain the eccentric anomaly, the firft part of which I believe to be new; the fecond, is fimilar to the method made ufe of by Sir Ifaac Newton in his Principia.—He firft affumes an arc, and then proceeds to find its error: but by the method which I have purfued, we proceed directly to the folution of the problem without any affumption, and therefore adhere more clofely to the principles of geometry.——The firft part of the operation will give the eccentric anomaly almoft fufficiently exact for any of the planets belonging to our fyftem; and the fecond which is very eafy, will produce a greater degree of exactnefs than is requifite for any of the bodies revolving round our Sun, the comets excepted. The method is as follows.

Let S, Fig. 9. Plate I. reprefent the Sun, and the arc AN the mean anomaly; join, SN—through the centre C; draw CP parallel to SN, and the angle ACP will be nearly the eccentric anomaly; and may be had by the following analogy.—From the log. tang^t. of half the mean anomaly, fubtract the difference of the logs. of the aphelion and perihelion diftances, the remainder will be the log. tang^t. of an angle, to which add half the mean anomaly, and the fum will be the angle ACP.———For an example, take the planet ♅.

Let the mean anomaly AN = 60° the half 30° log. tang^t.—9.7614394

Log. Aphelion dift. 6.3007704 }
Log. Perihelion dift. 6.2594052 } Deduct the difference —0.0413649, and

there remains log. tang^t. 9.7200754 which anfwers

anſwers to 27° 41′ 41″, to which add half the mean anomaly, and the
30
ſum - - 57 41 41 will be the angle ACP, which in this example will be about 3″ too ſmall, becauſe the right line ST, drawn at right angles to PC continued if neceſſary, and which is nearly the part to be deducted from the mean anomaly, will be ſhorter than the arc PN.—Then to find the value of ST:—ſuppoſe CA, or CP, to be equal to 1 or unity, then from the elements of ♅, SC will be equal to .04758735 and CP the radius being equal to an arc of 57°.29578,—SC will be equal to 2°, 7266,—then

As radius	- -	Log.	10.0000000
Is to SC,	2° .7266	Log.	0.4356115
So is the S. of TCS,	57° 41′ 41″	Log.	9.9269660
To ST, = 28 .3045	- -	Log.	0.3625775

60
18′ .2700
60
16 .2000. This arc of 2° 18′ 16″ being deducted from the mean anomaly will leave 57° 41′ 44″ for the eccentric anomaly corrected, and will be true within the $\frac{1}{5}$ part of a ſingle ſecond. If a greater degree of accuracy ſhould be requiſite, the corrected angle 57° 41′ 44″ which ſuppoſe to be ACO, muſt be uſed to obtain the value of SR, and that value applied as above. This correction will only be neceſſary in caſes where the orbits are very eccentric.

But as the planets do not revolve in circular, but elliptical orbits, the point O, in the arc AN, muſt be reduced to *r* the place of the planet in the curve of the ellipſe; which is the point cut by the right line OF drawn at right angles to AD.—The angle AS *r* will then be the co-equate, or true anomaly; and may be had by the following analogy.—From the log. tang^t. of half the eccentric anomaly, take the difference of the logs. of the aphelion and perihelion diſtances, and the remainder will be the log. tang^t. of an angle; to this angle add half the eccentric anomaly, and call the ſum U. Then to the log tang^t. of U, add half the ſum of the logs. of the aphelion and perihelion diſtances; from that ſum deduct the log. of the mean diſtance, and the remainder will be the log. tang^t. of the co-equate, or true anomaly.—For example

From half the eccentric anomaly 28° 50′ 52″ log. tang.		9.7410263
Deduct the diff. of the logs. of the aphelion and perihelion diſtances		0.0413649
The remainder 9.6996614 will be log. tang^t. of	26° 36′ 5″	
Add $\frac{1}{2}$ the eccentric anomaly -	28 50 52	
Call the ſum U - - .	55 26 57	

Then

Then to U* = 55° 26′ 57″	Log tang^t.	10 1620405
Add half the ſum of the log^s. of the aphelion and perihelion diſt.		6.2800876
		16.4421281
Deduct the log. of the mean diſt.		6 2805800
The remainder is the log. tang^t. of 55° 25′ 7″		10.1615481

The co-equate or true anomaly 55° 15′ 7″ is the meaſure of the angle AC *r*, and when deducted from the mean anomaly will leave the equation of the centre: as for example, 55° 25′ 7″ taken from 60° the mean anomaly uſed in the foregoing explanation the remainder 4° 34′ 53″ will be the equation of the centre anſwering to it.—The equation of the centre muſt be negatively applied while the planet is moving from the aphelion, to the perihelion, and *vice verſa*.

I am, Sir, with much eſteem,
Your real Friend,
ANDREW ELLICOTT.

To Mr. Robert Patterſon.

N°. IX.

Method of raiſing the common Logarithm of any Number immediately, by David Rittenhouse, *Preſident of the Society.*

Read Aug. 12, 1795.

THE logarithm of any number is the index of that power of 10 which is equal to the given number. This index will always be fractional, unleſs the number be diviſible by 10 without any remainder.

If the number be greater than 10, divide it by the higheſt power of 10 that will leave the quotient not leſs than 1. The index of that power is the firſt figure, or index of the logarithm. Divide 10 by the quotient ſo found raiſed to the higheſt power that will leave the new quotient not leſs than unity. Divide

* *Note.* When U exceeds 90° take its ſupplement and in that caſe deduct the reſult of the calculation from two right angles, and the remainder will be the true anomaly.

Fig. 2. P. 54.

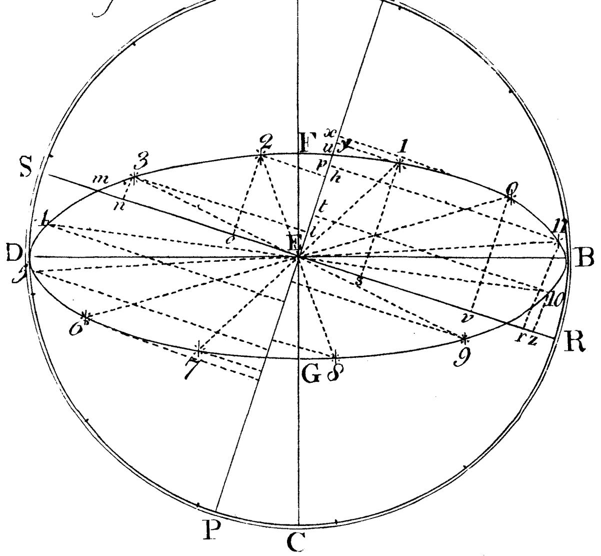

Fig. 1. P. 51.

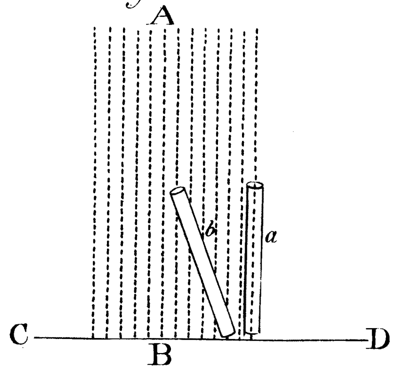

Fig. Θ P. 27.

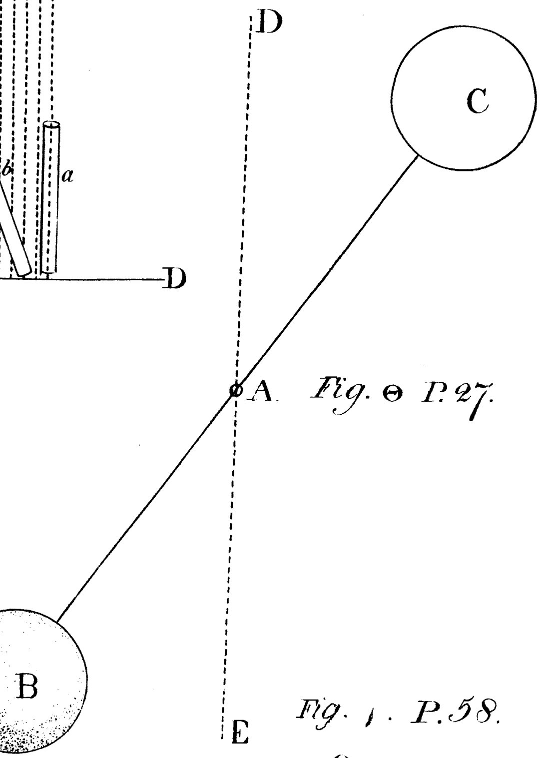

Fig. 3. P. 58.

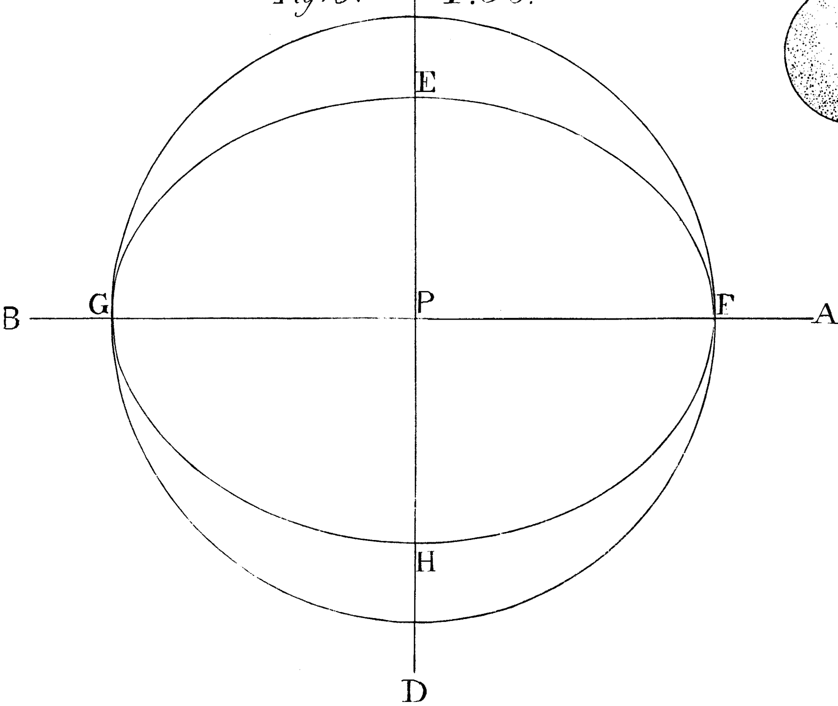

Fig. 4. P. 58.

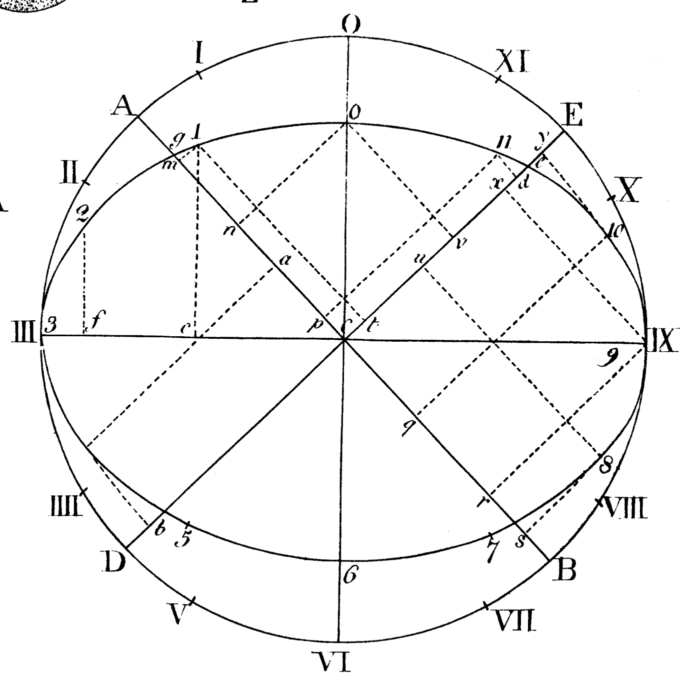

Fig. 5. P. 65.

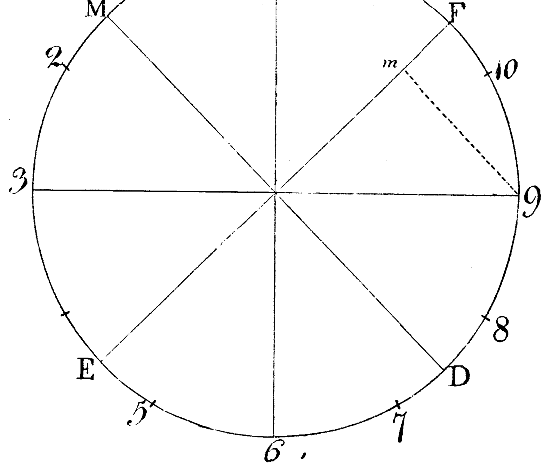

Fig. 9. P. 67.